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Green-Schwarz Anomaly Cancellation, World Sheet Instantons and Wormholes

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Abstract

We consider the breaking of the global conservation of gauge field charges which are commonly thought to survive the spontaneous breakdown of gauge symmetry brought about by Kalb-Ramond fields. Depending on the dilaton field and also the size of the compactifying space, the global charge breaking may take place due to world sheet instantons. In going to $3+1$ dimensions one could have a serious problem in order to produce the hierarchies between the quark and the charged lepton masses using the mass protecting charges with the Green-Schwartz anomaly cancellation. Various unnatural features of this type of models are discussed.

Key words: Green-Schwarz anomaly cancellation mechanism, Kalb-Ramond field, World sheet instantons, Wormholes
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1 Introduction

It has been believed some time ago that gauge symmetries with anomalies cancelled by the Green-Schwarz mechanism [1] in $9+1$ dimensions could be spontaneously broken in such a way that, after compactifying to four dimensions, a global symmetry inherited from the gauge symmetry could, in principle, survive, *e.g.*, [2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28,29].

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The survival of a global gauge symmetry – even after the gauge bosons have acquired a Higgs field induced mass – is quite mysterious because the vacuum is not gauge invariant under the gauge symmetry with constant gauge function (Λ = const.) due to the Kalb-Ramond field (which plays a major role in this phenomenon). Despite this, it seemed as though there remains a phase transformation symmetry for the fields carrying the family-dependent $U(1)_X$-charge when gauge symmetry is spontaneously broken in this remarkable way.

We emphasise in this article that, precisely this phase transformation symmetry is *not* a true symmetry if one takes into account the world sheet instantons. We shall argue below that if the compactifying space is of order of the fundamental scale, then the local and global gauge symmetry gets totally broken. However, it is unrealistic to compactify the space so close to the fundamental scale. The crucial point is that the effect of the world sheet instantons will be exponentially suppressed. In the case of very strong breaking (of the order of the fundamental scale) it would mean that we could not apply the Green-Schwarz anomaly cancellation mechanism in $3+1$ dimensions, *i.e.*, for the application to the large hierarchical Yukawa coupling constant structures.

On the other hand, if we let the compactifying scale be much below the fundamental scale, there is another odd feature: there is very light (Abelian) gauge particle from the fundamental scale point of view and correspondingly we get that the condition $F_{\mu\nu}\widetilde{F}^{\mu\nu}=0$ for not having anomalies is fulfilled identically. This would lead to a rather strange electrodynamics. If the validity of the usual $\mathrm{div}\vec{E}=j^0$ is not maintained, the dynamics could open the possibility for space-time foam causing the break down of the global charge conservation due to wormholes.

This article is organised as follows: in the next section, we review the Green-Schwarz anomaly mechanism, and in section 3 the world sheet instantons, and we also discuss the string coupling constant. Section 4 contains the various discussions including the suspected effects of wormholes. Finally, section 5 contains our conclusions.

2 Review of Green-Schwarz anomaly mechanism

Let us review the Green-Schwarz anomaly cancellation mechanism focusing on the Kalb-Ramond field in $9+1$ dimensions and then the application for the $3+1$ dimensions.

For the purpose of making phenomenological fits of the quark and (charged) lepton masses and mixing angles it is very useful to have some approximately conserved charges [30] (in addition to the gauge charges of the Standard

Model) so that most of the masses get suppressed due to the differences in quantum numbers of the right- and left-handed Weyl components. It is very attractive, and needed due to the effects of wormholes *etc.* [31], to let such mass suppressing charges be gauge charges. There are many gauge charges in superstring theory so such a picture is not unnatural in this theory. Working in $3+1$ dimensions one would at some level expect to obtain a $3+1$ dimensional field theory with gauge fields which could be described as renormalisable. That in turn would imply that the triangle anomalies resulting from the various chiral fermions in the effective $3+1$ dimensional model should cancel, *i.e.*, no violation of gauge symmetry would be caused. Otherwise this effective model would not be renormalisable. Now, however, it became very popular to use the inspiration from the superstring theory to suggest models in which this "usual" gauge anomaly cancellation does *not* take place. From the four dimensional point of view this avoidance, which is usually needed for renormalisation, gauge- and mixed anomaly cancellation conditions seems quite extraordinary: A certain coefficient field $b(x^\mu)$ in an expansion for the Kalb-Ramond field B_{MN} $(M,N=0,1,\ldots,9)$ in the $9+1$ dimensional theory, couples as an axion field. That is to say it couples via the Lagrangian density term of the form

$$\mathscr{L} = b(x^\mu)\, F_{MN}\, \widetilde{F}^{MN} + \cdots \quad , \tag{1}$$

where $\mu = 0,1,2,3$.

In the superstring theories (type II and Heterotic strings) there is a Kalb-Ramond anti-symmetric tensor field with two indices on the potential $B = B_{MN}\, dx^M \wedge dx^N$ and three on its field [1]

$$H = dB + \omega^0_{3Y} - \omega^0_{3L} \quad . \tag{2}$$

Here the three forms ω^0_{3Y} and ω^0_{3L} are given by [32,33].

In these theories there is a very sophisticated way of cancelling the gauge, gravitational and various mixed anomalies, first by having the right number of chiral fermions but in addition some terms, which are gauge non-invariant when alone, in the action for zero mass particles are used,

$$\begin{aligned}
S_1 &= c \int \left(B\, \mathrm{tr} F^4 + \frac{2}{3}\, \omega^0_{3Y}\, \omega^0_{7Y} \right) \quad , & (3)\\
S_2 &= -c \int \left[\frac{1}{32}\, B \left(\mathrm{tr}\ R^2 \right)^2 + \frac{1}{8}\, B\, \mathrm{tr}\ R^4 + \frac{1}{12}\, \omega^0_{3L}\, \omega^0_{7L} \right] \quad , & (4)\\
S_3 &= c \int \left(\frac{1}{8}\, B\, \mathrm{tr}\ R^2\, \mathrm{tr}\ F^2 + \frac{1}{48}\, \omega^0_{3L}\, \omega^0_{3Y}\, \mathrm{tr}\ R^2 - \frac{1}{24}\, \omega^0_{3Y}\, \omega^0_{3L}\, \mathrm{tr}\ F^2 \right.
\end{aligned}$$

$$- \frac{2}{3}\,\omega^0_{3L}\,\omega^0_{7Y} + \frac{1}{12}\,\omega^0_{3Y}\,\omega^0_{7L}\Big) \quad , \tag{5}$$

to cancel the remaining part of the anomalies. Here c is numerical constant [34,32].

In order to get chiral fermions – as is phenomenologically required to obtain the Standard Model – it is needed to break the parity symmetries that only makes reflections in the compactifying dimensions (for instance by having non-zero magnetic field in the extra dimensions). One may typically make use of Calabi-Yau spaces as the 6-dimensional compactifying space. For pedagogical reasons, just to illustrate the idea we may in the present article think of a compactifying space being the cross product of three spheres, each of topology S^2 and each with a magnetic field on them corresponding to a "magnetic monopole in the centre of the S^2 sphere". Let us imagine that the equations of motions have led to that the vacuum has S^2 rotation invariant fields on the different S^2's. Then we may symbolically use these rotational invariant [1] field strength $\langle F_{67}\rangle$ *etc.* The first term in integrand of S_1 (Eq. 3) which by itself gauge breaking, will contain a contribution of the form

$$\mathscr{L} = B_{45}\ \langle F_{67}\rangle\ \langle F_{89}\rangle\ F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\ + \cdots\ . \tag{6}$$

Imagine expending the variation over the S^2 sphere (*i.e.*, x^4 and x^5 dependence) on "spherical harmonics" or "eigenfunctions". Suppose we arranged one of spherical harmonic or eigenfunction to be dominant in the weakly exited state. We describe the effective four dimensional theory by means of the expansion coefficient $b(x^\mu)$ to this term:

$$B_{45}(x^\mu, x^4, x^5) = f(x^4, x^5)\, b(x^\mu)\ . \tag{7}$$

Really we could define such $b(x^\mu)$ by integrating the two form B over a homotopically non-trivial 2-cycle. This would then require that we imposed other terms in the expansion of B_{45} to be restricted to zero. Taking the magnetic fields in the compactifying dimensions as constants we end up with an effective term in the four dimensional Lagrangian density which up to the over all constant is of the form (1). From the kinetic term for the Kalb-Ramond field,

$$- \frac{3\kappa^2}{2g^4\varphi^2}\, H_{MNP}\, H^{MNP}\quad , \tag{8}$$

where κ is the gravitational coupling constant, φ the dilaton field, g the Yang-Mills gauge coupling constant in the Lagrangian density, we obtain a kinetic

[1] This S^2 rotation symmetry is, we have in mind, that the topological S^2 is represented by a sphere which then has the symmetry under $SO(3)$ rotations (about a point outside the sphere) with respect to the fields assumed to be present.

term for the coefficient field $b(x^\mu)$. Due to the ω_{3Y}^0 term in Eq. (2) it comes together with an Abelian part of the Yang-Mills potential in an expression of the form

$$\frac{1}{2}\, m^2\ (\partial_\mu b - A_\mu)^2 \quad . \tag{9}$$

This is a gauge invariant combination for the b-field gauge transform

$$b \to b + \Lambda \ , \tag{10}$$

while

$$A_\mu \to A_\mu + \partial_\mu \Lambda \ , \tag{11}$$

where Λ is the gauge function for an invariant $U(1)$ subgroup of the left over symmetry group, not spontaneously broken.

For simplicity we imagine that the presence of the extra dimension fields $\langle F_{67}\rangle$ $\langle F_{89}\rangle$ represents a break down to a subgroup which still contains at least one invariant Abelian subgroup called $U(1)_X$. Then we may concentrate on the gauge field associated with this subgroup $U(1)_X$ and denote the gauge function for it as Λ. We shall discuss a rather extraordinary behaviour of the theory – from the four dimensional point of view – with the axion field $b(x^\mu)$ (see also Sec. 4.1).

Suppose that $b(x^\mu)$ does not quantum fluctuate so widely that it totally looses an expectation value. This means that there is a spontaneous break down of the gauge symmetry for $U(1)_X$. Since even the constant Λ gauge transformation is spontaneously broken due to the additive transformation property of b, the spontaneous breaking situation is just like that of the Higgs case. However, that means, one would expect that particles – such as fermions carrying $U(1)_X$-charge quanta – would be able to make transitions into (sets of) particles with a different number of such charges (together). At this point, however, one has often found the belief that the global symmetry and the Noether conservation of the charged particles is *not* violated. In the perturbative approximation this belief is well-granded. We will discuss this question in the following in non-perturbative approximation.

3 World sheet instantons and the Fayet-Iliopoulos D-term

Although at first it seems as if there is no way to cause the global $U(1)_X$-charges on particles to be created or annihilated, it was shown in [35,36] that

such a violation of the charge was indeed occurring due to world sheet instantons. These "world sheet instantons" refer to the tunnelling of a string so as to have a "time track" during tunnelling which encloses in our simple scenario the S^2 involved with the B_{45}. In the real general case we should have the tunnelling go around a 2-cycle homotopical to the 2-cycle(s) used for extracting b from B.

The important point for the present discussion is as follows: When such a world sheet instanton exists, there are zero modes for the fermions (as well as bosons) which have $U(1)_X$-charge. These zero modes cause the $U(1)_X$-charge to change. In this anomalous way – similar to the QCD-instanton – the global charge gets also violated after all. According to [35,36] this is much more natural than not having the global $U(1)_X$-charges broken.

In reality the effect of this zero-mode effect is described by an effective Lagrangian term

$$e^{-ib}\, Q = e^{-ib} Q_1 \cdot Q_2 \cdots Q_n \ . \tag{12}$$

Here the $Q_1, Q_2, \ldots, Q_n$ are various $U(1)_X$-charged fields and the product $Q = Q_1 \cdot Q_2 \cdots Q_n$ could, for instance, be $Q = \overline{\psi}\psi$ where ψ is a field for which $U(1)_X$ has the role of a chiral mass protecting charge. The whole term is to be multiplied by the amplitude of the world sheet instanton. The factor e^{-ib} comes from the exponentiated string action Eq. (30), which may cause the damping and the factors $Q = Q_1 \cdot Q_2 \cdots Q_n$ are needed to make the whole term gauge invariant.

In supersymmetric theories, where these models are usually considered, such an axion field must occur together with a corresponding field in a complex combination. For instance the anomalous Fayet-Iliopoulos D-term was studied [4] in the context of the heterotic string theory: They consider a dilaton chiral supermultiplet, Φ, adding to the Kähler potential a part, $K_d\left(\Phi + \overline{\Phi}\right)$, which transforms under the $U(1)_X$ gauge group as

$$\delta\Phi = i\alpha \ . \tag{13}$$

The Kähler potential is with appropriate vector kinetic term

$$K = -\ln(\Phi + \overline{\Phi})\,, \qquad f = \Phi\,, \qquad \Phi \equiv \phi^{-2} + ib\,. \tag{14}$$

The gauge coupling constant thus depends on the dilaton, $(\mathrm{Re} f)^{-1} = \phi^2$. The theory also has an axion coupling proportional to $b\, F_{\mu\nu}\, \widetilde{F}^{\mu\nu}$. With the shift transformation of the axion field under $U(1)_X$ gauge group (see Eq.(13)), this term serves to remove the anomaly proportional to $F_{\mu\nu}\, \widetilde{F}^{\mu\nu}$.

The D-term potential is using Eq. (14)

$$\frac{1}{2}(\mathrm{Re}f)^{-1}\left\{K_d'\left(\Phi+\overline{\Phi}\right)\right\}^2=\frac{1}{8}\phi^6 \ . \tag{15}$$

We should emphasise that in the weak coupling limit, where dilaton field goes to zero ($\phi^2 \to 0$) the Fayet-Iliopoulos D-term vanishes, *i.e.*, there is a supersymmetric minimum.

If we contrary to the just given argumentation assumed that after all non-zero Fayet-Iliopoulos D-term were stabilised – *i.e.*, there were at least the metastable minimum for non-zero ϕ – then for the weakly-coupled heterotic string according to Ref. [5] with $g_s = \phi^2$, there is a Fayet-Iliopoulos D-term given by

$$\xi_{GS}=\frac{g_s^2\,\mathrm{Tr}q}{192\pi^2}M_{Pl}^2 \ , \tag{16}$$

where M_{Pl} is the Planck mass and $\mathrm{Tr}q$ is the sum of $U(1)_X$-charges.

Thus the potential energy becomes

$$V_D=\frac{1}{2}\,g_s^2\xi_{GS}^2=\frac{(\mathrm{Tr}q)^2}{73728\pi^4}\,g_s^6\,M_{Pl}^4 \ , \tag{17}$$

so that V_D is obviously negligible at very small dilaton field even if one wishes to have a model in which $\mathrm{Tr}q$ is non-zero, a typical value of $\mathrm{Tr}q \sim 10^2$ to 10^3 (see *e.g.* [9]).

According to [12] the D-term (16) may induce some (non-zero) expectation value of a scalar field called θ, and thus cause a spontaneous breakdown of the global (part) of the $U(1)_X$ symmetry. Supposing no other scaler field break this group, then the non-zero vacuum expectation value of θ may be made a connection with the Fayet-Iliopoulos D-term in following way:

$$|\langle\theta\rangle|=\sqrt{-\frac{\xi_{GS}}{X_\theta}}=\sqrt{\xi_{GS}} \ , \tag{18}$$

where the $U(1)_X$-charge of the Higgs field θ, X_θ, is taken to be -1. In this way one could obtain an expansion parameter for the fermion mass matrices from ξ_{GS}: In the case of ξ_{GS} being not too large (see Eq. (16)), in other words, g_s is not too small and the sum of $U(1)_X$-charges ($\mathrm{Tr}q$) is of order 10^2 to 10^3, we could use such a mechanism to produce a good order of magnitude for the breaking of the global charge conservation for $U(1)_X$. The $U(1)_X$ symmetry

would be broken one or two orders of magnitude below the fundamental scale (Planck or string scale, depending on model). This situation could be using the fundamental scale as the Planck scale,

$$
\begin{aligned}
\epsilon \equiv & \frac{|\langle\theta\rangle|}{M_{Pl}} \\
= & \sqrt{\frac{\xi_{GS}}{M_{Pl}^2}} \sim 0.2 \ ,
\end{aligned} \tag{19}
$$

i.e., an expansion parameter for the fermion mass matrices which is identified with the Cabibbo angle.

If we have identified the string coupling constant, g_s, with a dynamical field – the dilaton field ϕ^2 – the ground state will be found by adjusting this coupling g_s to be zero, which obviously means the disappearance of the global charge breaking effect. In this situation, which is the expected one, we have thus at first no complain from world sheet instantons about the possibility hoped for in literature of having the global charge totally conserved although derived from a Higgs gauge symmetry. However, if the string coupling constant g_s is really going to zero, then the models in which this happens have zero string coupling constant, and they become *totally free*, since in string theory all interactions are finally derived from the string interaction g_s. Unless one can somehow live with a tiny breaking of supersymmetry at very high energy scale allowing a small but non-zero g_s, the models avoiding a Fayet-Iliopoulos D-term would become totally free as string theories!

The wish that the adjustment lets the Fayet-Iliopoulos D-term vanish by adjusting the ϕ^2 to be zero is further supported by the often favored phenomenological call for supersymmetry not to be broken except at low energy scales, typically close to 1 TeV, because it can be of help for the hierarchy problem. From the point of view of the high scale of energy, where we *a priori* have the Higgsing of the $U(1)_X$ group, a phenomenologically useful supersymmetry breaking scale would be tremendously small and essentially count as zero. This argument further disfavours models which would have troubles due to the Fayet-Iliopoulos D-term. But if the problem is solved by adjusting to the supersymmetry which is only extremely weakly broken by having the string coupling almost zero, then the type of model is even more problematic, because it lacks the interaction altogether.

As a resume of the above discussion let us compare the two logical possibilities concerning the size of the string coupling constant g_s of being of order unity $g_s \approx \mathscr{O}(1)$ or sufficiently small ($g_s \approx 0$) both being considered in the presence of a non-trivial Green-Schwarz anomaly cancellation so that the Fayet-Iliopoulos D-term becomes non-zero unless $g_s = 0$:

(1) Consistence
The possibility $g_s \approx \mathcal{O}(1)$ is strictly speaking inconsistent because the Fayet-Iliopoulos D-term (Eq. (16)) drives the g_s, which is effectively dynamical (related to ϕ), to zero so that only $g_s \approx 0$ is consistent.

(2) The expansion parameter ϵ suitable for small hierarchy?
A reasonable sized $g_s \sim 1$ could give a good expansion parameter ϵ, however, if $g_s \approx 0$ of course ϵ becomes exceedingly small and not useful for fitting the small hierarchy.

(3) Supersymmetry for hierarchy problem phenomenology?
$g_s \approx \mathcal{O}(1)$ leads to extremely high scale supersymmetry breaking by the Fayet-Iliopoulos D-term such that the supersymmetry is of no use for solving the hierarchy problem. (explain any weak scale supersymmetry phenomenology for that matter.) While for a very small $g_s \approx 0$ one may get supersymmetry accurate enough for hierarchy problem purposes.

(4) Freeness of the whole string theory
For $g_s \approx 0$ the string theory becomes free while for $g_s \approx \mathcal{O}(1)$ there are interactions.

(5) Conservation of the global charges of $U(1)_X$
There are two possibilities with g_s in the relative large range which we should mention:

- *(a)* g_s is large: in this case the world sheet instantons and the Fayet-Iliopoulos D-term may be too large so that the breaking, which they cause, would also be too large (*i.e.* the expansion parameter ϵ). In this case $U(1)_X$-charge cannot be used to the small hierarchy problem of the fermion masses.
- *(b)* g_s is not too large: in this case g_s could give a good expansion parameter ϵ (Eq. (19) and see Sec. 4.3). Then one could hope for the desired breaking of global charge *without* invoking further breaking mechanisms.

In the case when g_s is really small the global charge is very well conserved, but one can imagine it broken by other means so that it would be no problem.

(6) Higgsing
The Higgsing of the local part of the gauge group uses the Kalb-Ramond field and that works independent of the size of g_s (unless of course the whole theory becomes free that one can conclude that nothing happens at all).

(7) Anomaly cancellation for $U(1)_X$ from four dimensional point of view

- *(a)* a strong g_s: In this case the world sheet instanton breaking would be strong and there would essentially be not even a global charge approximately conserved.
- *(b)* a bit weaker g_s: there is an approximately conserved global charge which does not have the cancellation by triangle diagrams which is required for a gauge charge.

4 Discussions

4.1 The extraordinary properties of the four dimensional effective theory

The four dimensional model is derived from an although non-renormalisable – as all theories in ten dimensions – then at least gauge invariant theory. It is therefore surprising that it does not satisfy the usual conditions on numbers of fermion species and their charges needed for the anomaly cancellation. It is the reason why there is the Wess-Zumino term in Eq. (2). However, how does that remove the need for anomaly cancellations?

One might wonder how the situation of the anomaly cancellation would be, if we had that the mass scale m of the $U(1)_X$-photon (Eq. (9)) were very low compared to the scale of energy at which we consider the situation. From the point of view of such a high energy scale compared to m, it would seem that the kinetic term for the axion field, b, were very close to having zero coefficient, *i.e.*, an auxiliary field. The effect of integrating out b functionally would be to produce a functional δ-function with the effect of imposing the constraint

$$F_{\mu\nu}\,\widetilde{F}^{\mu\nu} = 0 \ . \tag{20}$$

With such a constraint imposed on the gauge field it would of course be no wonder if one gets no anomalies. In fact it would mean that the anomaly

$$\partial_\mu\, j^\mu \propto F_{\mu\nu}\,\widetilde{F}^{\mu\nu} \tag{21}$$

had been constraint to be zero. Such a constraint will lead to interactions between photons which are of course in the next approximation in m^2 understandable as due to exchange of the b-particles, *i.e.*, $\gamma\gamma \to \gamma\gamma$. However, notice that diagrams, like Fig. 1, have the b field propagator which has the contribution of a factor m^{-2}. This propagator contribution is tremendous compared to p^{-2} since p is in the range (orders of magnitude) above the $U(1)_X$-photon mass scale, m, *i.e.*, $\gamma\gamma$-scatterings have extremely strong interactions. Therefore, they may be able to provide the constraint forces which uphold the constraint Eq. (20) (or Eq. (28)).

4.2 Equations for the four dimensional effective theory

We have an interacting $U(1)_X$-photon theory with the Lagrangian

$$\mathscr{L} = -\frac{1}{4}\,F_{\mu\nu}\,F^{\mu\nu} + \frac{b}{16\pi^2}\,F_{\mu\nu}\,\widetilde{F}^{\mu\nu} \ , \tag{22}$$

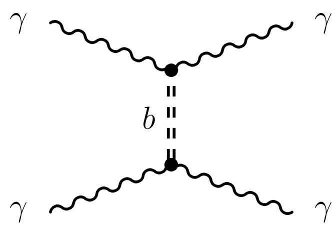

Fig. 1. Feynman diagram of the photon-photon scattering by b field exchange.

which in addition has the term (9) when we do not consider m so small that we can ignore the term in Eq. (9).

The equations of motion become, in addition to Eq. (20) derived by varying b,

$$\partial_\mu F^{\mu\nu} = \frac{1}{4\pi^2} \, (\partial_\mu b) \, \widetilde{F}^{\mu\nu} \quad . \tag{23}$$

Including charged matter and noticing that the no-matter terms in the Euler-Lagrangian equation can be written in a form more familiar, we obtain

$$\begin{aligned} \partial_\mu F^{\mathrm{Red}\ \mu\nu} &= \partial_\mu \left(F^{\mu\nu} - \frac{b}{4\pi^2} \, \widetilde{F}^{\mu\nu} \right) \qquad (24) \\ &\equiv J^\nu \quad , \qquad (25) \end{aligned}$$

where J^ν is the "matter current", and we may define the short hand notation

$$F^{\mathrm{Red}\ \mu\nu} = F^{\mu\nu} - \frac{b}{4\pi^2} \, \widetilde{F}^{\mu\nu} \quad . \tag{26}$$

For future discussions it is convenient when we express the equation of motion (Eq. (23)) with the electric field, $\vec{E}$, and magnetic field, $\vec{B}$,

$$\mathrm{div} \vec{E} = \frac{1}{4\pi^2} \, \mathrm{div} \left(b \, \vec{B} \right) \quad , \tag{27}$$

hereby we have applied $\mathrm{div} \vec{E}^{\mathrm{Red}} = 0$.

A mathematical point worth noticing in connection with the somewhat unusual electrodynamics, which we discuss here, is that the condition (20) can be shown by trivial algebra to imply that even

$$F_{\mu\nu} \, \widetilde{F}^{\nu\rho} = 0 \quad , \tag{28}$$

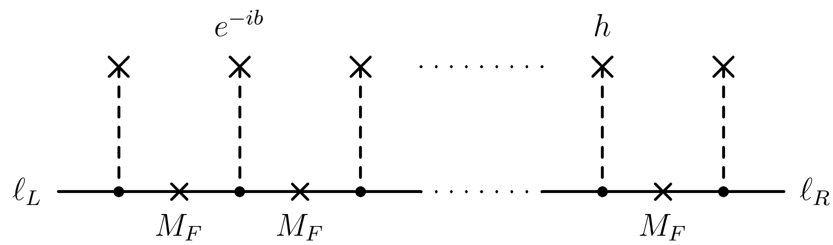

Fig. 2. Feynman diagram for fermion mass term. The dashed lines with crosses symbolise of different Higgs fields (h) or the world sheet instanton (e^{-ib}). M_F is denoted as the fundamental scale.

for all combinations of the indices of μ and ρ.

4.3 Order of magnitude possibilities

Above, we have reviewed an anomalous horizontal $U(1)_X$ model using Fayet-Iliopoulos D-term to cause a spontaneously break down of $U(1)_X$-charge. This effect would arise if the dilaton field ϕ were non-zero which is, however, not achievable due to supersymmetry being driven to be an exact symmetry. However, if we have a non-zero dilaton field, we also have world sheet instanton effects breaking the global charge conservation using Green-Schwarz anomaly cancellations. Thus, this suggests to use this instanton effect in stead of a Fayet-Iliopoulos D-term. Could this effect be adjusted for a phenomenologically reasonable global charge breaking of order of the Cabbibo angle ϵ?

Baring a totally mysterious cancellation of various contributions from different world sheet instantons the order of magnitude of the $U(1)_X$ breaking by world sheet instantons is estimated as the exponent of the supersymmetry associated term to the field b, which makes up a complex field. The possibility of surprising cancellations would have *a priori* to be ignored [9] if it were not for the findings that this indeed easily can occur [37]. Let us divide the discussion into the following possibilities:

(1) All the quantities, including $g_s = \phi^2$, are very strictly of order unity and the breaking of the charge conservation is also of order unity in spite of the fact that it is exponentially suppressed – as an instanton tunnelling effect. In this philosophy the charge conservation is strongly broken: Let us then imagine that the mass or the effective Yukawa coupling for a quark or a charged lepton is obtained via a chain diagram (Fig. 2) in which a series of fundamental scale vector coupled fermion propagators are linked by Higgs fields or world sheet instanton caused transition symbols. If the strength of the charge violating world sheet instantons is of just the same order of magnitude as the (typical) fundamental scale fermion masses, then

there will be no suppression, and the $U(1)_X$-charge considered will be of no help in explaining the suppression of some effective Yukawa couplings (at experimental scales) compared to others. However, taking "everything" especially the compactifying space dimensions to be very close to unity in "fundamental" units, such that even exponents are accurately of order unity is presumably not likely to be true.

(2) The other possibility is that there are some quantities which cannot be considered order unity in the very strong way discussed under point *(1)*. The compactifying dimensions turn out to be the quantities of importance for the strength of the $U(1)_X$-charge violation. Let it be clear that there are really two scales of breaking of the $U(1)_X$ symmetry to be discussed:

(a) There is the $U(1)_X$-photon mass scale, *i.e.*, the mass m of Eq. (9).

(b) The mass scale M_V that appear as the mass obtained for the fermions which are mass protected *only* by the $U(1)_X$ and get their mass via the world sheet instanton caused term (12) in the case of $Q = \overline{\psi}\,\psi$. It is this mass scale M_V which divided by the "fundamental" fermion masses M_F gives the suppression factor $\epsilon = M_V/M_F$ which is used as a factorisation parameter for the fermions mass matrices.

We discuss in the following the variation of the scales of breaking *(a)* and *(b)* above: The mass square factor in Eq. (9) goes back to the term (8) in as far as the b field is a coefficient on a term in B which in turn has its derivatives go into H in Eq. (8). It is remembered that the function multiplying b to get the B_{45} contribution must be normalised so that a shift in b by 2π,

$$b \to b + 2\pi \ , \tag{29}$$

corresponds to adding to $\int_{S^2} B$ the shifting by a single monopole flux field through the cycle S^2 shifted by $\Lambda = 2\pi$. If all the couplings are taken to be of order unity, one finds that scaling the dimensions of the 2-cycle as the typical compactifying space dimension, R, squared R^2. The area of the 2-cycle is proportional to R^2. This means that that $m^2 \propto R^{-2}$. Thus we see that the mass scale of the $U(1)_X$-photon goes as R^{-1} where R is really the length scale of the two-cycle.

The tunnelling suppression amplitude of the world sheet instanton [38,37] is

$$\exp\left(-\frac{A}{2\pi\alpha'} + i\int B\right)\frac{\mathrm{Pfaff}'\mathscr{D}_F}{\sqrt{\det'\mathscr{D}_B}} \ , \tag{30}$$

where α' is the Regge slope, Pfaff is the Pfaffian, and $\mathscr{D}_F$ and $\mathscr{D}_B$ are kinetic operators for the bosonic and fermionic fluctuations, respectively. The "$\,'\,$" on the Pfaffian and determinant denotes that the zero modes are to be omitted. Moreover, A is the area, which of course $A \propto R^2$ again, with R being the R

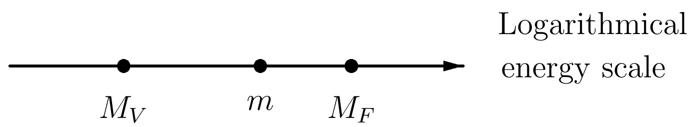

Fig. 3. The different energy scale. M_F is the fundamental scale. m is $U(1)_X$-photon mass. M_V is $U(1)_X$ breaking scale. Suggestive rations: $M_F/m \approx 1.5$, $M_F/M_V \approx 3$.

relevant for the two-cycle.

Introducing a fundamental mass scale, M_F, we thus have the scale at which the $U(1)_X$-charge conservation violates

$$M_V \approx M_F \, \exp\left(-R^2 \; M_F^2\right) \quad . \tag{31}$$

This crude estimate assumed g_s to be of order one.

Now, we may go into crude phenomenology, still taking g_s of order unity. In theories with compactified extra dimensions it is quite natural to take this as being the reason for the fine structure constants being weak, compared to the "self-dual" strength (defined as the value of fine structure constant which makes it equal to the corresponding monopole coupling constant, associated by the Dirac relation). The self-dual strength, $\widetilde{\alpha}_{U(1)_X}$, is approximately 1/2 since $\alpha_e \, \alpha_g = 1/4$ for a formal monopole α_g. From this point of view we can claim that a typical, say GUT, coupling of order $\alpha \approx 1/25$ is weaker by a factor ≈ 12 than the Abelian self-dual. Though this is exaggerating and should be corrected at least by the factor 3/5. Roughly taking anything of this order, we would now expect that R measured in "fundamental units" would be of the order $R \approx \sqrt[6]{12} \simeq 1.5$ (see Fig. 3). This would mean $m \sim M_F/1.5$ and $M_V^2 \sim M_F^2 \exp(-1.5^2)$, *i.e.*, $M_V \sim M_F/3$, a very useful suppression factor indeed. This is namely a typical order of magnitude for an ϵ with which one fits the fermion mass spectra [8,9].

In this way it looks very promising to get very naturally a good scale for violation strength for phenomenological fitting. We can say that, strictly speaking, a scale R of the compactifying dimensions just 1.5 times bigger than the fundamental length scale M_F^{-1} is so close to being of order unity, that there is hardly any call for any special explanation for this "deviation" from it being the fundamental size. That we can notice some small numbers in both the fine structure constants and in the suppressed violation of the $U(1)_X$-charge is due to respectively the 6 dimensional compactifying space and the exponentiation because of the world sheet instanton effect needed.

The energy scale gap in which we have the funny electrodynamics with the constraint $F_{\mu\nu}\widetilde{F}^{\mu\nu}=0$ is in the just sketched scenario (with $g_s \sim 1$) reduced to a scale factor in the 1.5 region. That is such a very small range that one would hardly be able to claim that anything strange will happen.

4.4 Wormhole discussion I

Independent of whether world sheet instantons do or do not break global charge, there is another mechanism threatening the global charge conservation: the effect of wormholes.

We shall in fact show below in Subsec. 4.5 that our estimation of the effects of gravitational wormholes present in the vacuum (or similar space-time foam effects) *will give rise to a significant violation of the global charge – in the four dimensional theory – with the non-trivial Green-Schwarz anomaly cancellation, although it has resulted from a gauge charge* and thus at first might have been suspected of not being violated by space-time foam effects.

Over most of the energy scale (logarithmical counted, see Fig. 3) between the fundamental scale and the $U(1)_X$ violation scale M_V, we have a globally conserved charge for $U(1)_X$ while the gauge particle "already" had its mass at $m \sim R^{-1}$.

Now, however (see Subsec. 4.5), according to arguments in Ref. [31] charges which are not gauge protected may disappear into wormholes or baby universes. Therefore, one may speculate whether such a charge conservation which is unprotected by gauge fields will not get its conservation spoiled by the Wheeler space-time foam. Actually such effects of breaking the global charge are expected at energy scales smaller than m. But we will see in Subsec. 4.5 that wormholes at even higher energy scales than m shall provide such breaking when the monopoles are used. The wormholes of low energy (smaller than m), *i.e.*, of large (length) are suppressed exponentially with some exponent related to m/M_F. In fact we would (naively) estimate that for a space-time foam ingredient such as a baby universe of size m^{-1} (in length) we would have an action of order M_F^2/m^2, and a suppression factor of $\exp(-M_F^2/m^2) \approx \exp(-R^2M_F^2)$. It happens under the assumption $g_s \approx 1$ to be just the same order of magnitude – in the exponent suppression factor – as the one present in the world sheet instanton suppression factor. For this coincidence to occur it was quite crucial that the estimate just used for the baby universe action was of the form $\int_C R\,\sqrt{g}\;d^4x$ (C is baby universe tube) as being obtained by use of Einstein-Hilbert action and not $\int_C \sqrt{g}\;d^4x$ which would have been the case if the cosmological term in the action were significant here. That is to say, we used that the cosmological constant is zero, but

at these scales one is closer to the (running) cosmological constant which is relevant for short distance and which may have another value than the long distance one which is practically zero. In any case even if the cosmological constant were relevant, it would suppress the baby universes at the scale m even more.

The space-time foam non-conservation is expected to be at most what the world sheet instanton effect would have been if the g_s were of order unity, contrary to true expectation. Now, however, since we truly do not expect the world sheet instantons to provide breaking then the wormhole effects could easily take over as the dominant effect. Rather we should say that we do expect an appreciable wormhole breaking (see the argument below).

It could be that the world sheet breaking effect could actually dominate even in the case of $g_s \approx 1$, but the result would in this case not be so different with respect to the order of magnitude of the (exponent of the) effect, and thus the conclusion would be the same. However, in the case that there is no significant world sheet instanton effect, the wormholes can very well work and completely and dominantly break the charge conservation.

4.5 Argument for wormhole effect violation of the global part of the $U(1)_X$-charge – Wormhole discussion II

In this subsection we will prove for that the $U(1)_X$-charge is violated as a global charge due to the wormhole effects.

In the foregoing subsection we saw that one could give immediate arguments both in favour of and against the global $U(1)_X$-charge being violated by the wormhole or space-time foam effects. In this subsection we want to deliver an argument which shows that indeed *the global $U(1)_X$-charge is broken due to the wormholes.* We should, however, stress that this breaking is exponentially suppressed but *only with the suppression corresponding to the scale of the Higgsing m*, which is essentially the compactification scale. In the realistic models this is a rather high energy scale and the breaking from wormholes is thus expected to be quite large. Compared to what one gets from world sheet instantons, which do not work after all in the case of g_s being of order one, this could be much bigger than the latter.

It is most easy to organise the non-conservation of the Coulomb field for the $U(1)_X$-charge by use of virtual wormhole entrances with magnetic fluxes radiating out. These form effective virtual magnetic monopoles in the vacuum. The reason that it is profitable with monopoles violating the charge conservation for the $U(1)_X$-charge is that we indeed can derive some formulas for the variation/development of the $U(1)_X$-electric-charge relating it to the variation

of b on the sites of the monopoles (see Eq. (27)),

$$\partial^0 \mathrm{div}\vec{E} = \frac{1}{4\pi^2}\,\partial^0 \mathrm{div}\left(b\vec{B}\right) \quad . \tag{32}$$

Since $\mathrm{div}\vec{E}$ is the charge density this formula tells us that, for instance, when a monopole is present, the variation rate of the charge $\partial^0\mathrm{div}\vec{E}$ contains a term $(\partial^0 b)\,\mathrm{div}\vec{B}$ on the site of a monopole whenever b varies, *i.e.*, $\partial^0 b \neq 0$. The monopoles to be used here do not have to be genuine monopoles. They could be entrances to wormholes with magnetic flux going through [39]. We would not even have to use such genuinely existing monopolic wormhole entrances. It is rather sufficient to consider entrances which are virtually present in the vacuum. We shall imagine that there are many such wormholes virtually present with magnetic flux and that the entrances give rise to interactions with the various fields in the theory. We assume that interactions can be described by effective terms in the Lagrangian density. At first they are only at the places where the entrances to the wormholes are. We shall, however, integrate over all the possible positions or movements for the wormholes – as part of the functional integration of Feynman path integral. This has the implication that one can achieve naturally that such a model of wormholes can become effectively translational invariant. In fact one has to integrate over the positions – of the wormhole or baby universe entrances – with a translational invariant measure. That can actually be supported by a Heisenberg-inequality type argument using that at least baby universes cannot transport energy and momentum, because the information about these quantities is safely stored in the gravitational field at long distances from a (supposed to be) little baby universe. Even for wormholes it is reasonable to integrate over all positions with a translational invariant measure.

Since it is clearly possible that any sort of particle could be scattered into a wormhole, the effective Lagrangian density contribution from the entrance to a wormhole can contain terms annihilating or creating any combination of particles. Thus any combination/product of fields is *a priori* possible and will come with some coefficient in the effective wormhole and baby universe Lagrangian. Now, however, there can be processes that cannot really take place due to Coulomb fields left behind. By this we mean that if we propose terms violating gauge symmetries for charges with light gauge particles associated there remains information outside. In fact there will be a Coulomb field left, carrying the information about the charge of the particles which have gone into the wormhole. Even if the particle goes deeply into the wormhole and maybe even out somewhere else far away, there will remain electric flux lines exiting from the entrance of the wormhole and even if no appropriate – may be different – particle is pulled out the entrance the wormhole itself will behave as a charged particle. In this way we can only have effective Lagrangian density terms conserving the gauged charges corresponding to gauge particles with

Compton wave lengths which are long compared to the wormhole sizes.

Really what matters is, whether the gauge field around the wormhole can keep the information about what went into it. In the case of the conserved $U(1)_X$-charge the div$\vec{E}$, that should have ensured the stability of the Coulomb field, does not correspond to a conserved current as in the usual electrodynamics. It is rather the current corresponding to the $F^{\mathrm{Red}\,\mu\nu}$ (Eq. (26)) that is conserved. In fact we have just seen that if the axion field b varies on the sites of the virtual monopoles we can/will have that div$\vec{E}$ and thus the charge varies.

In this way it should now be *allowed* to have Lagrangian density terms due to the monopolic wormholes violating the $U(1)_X$. Once such terms are allowed they are expected to be there and we will generate masses for particles which are only mass-protected by $U(1)_X$ group.

Once we have the symmetry strongly broken at the Planck scale which is now expected there will no longer be a sign of the conservation and thus also no problem with the anomalies. The symmetry seems to be dynamically broken – not only spontaneously – because the effective Lagrangians representing the wormhole and other space-time foam effects really have to be interpreted as dynamical breaking. We must also expect that it is rather impossible to keep the $U(1)_X$-photon mass m to be light compared to M_V under such conditions.

We conclude that seriously taking wormholes into account in this way, it results that the Green-Schwarz anomaly cancellation scheme *does not work* in $3+1$ dimensional limit.

What happens is that the very strong constraint ensuring forces due to very large b propagators lead to the possibility of Coulomb fields around a wormhole entrance modified with time. This modification possibility in turn allows the effective Lagrangian density corresponding to the absorption of the $U(1)_X$-charges into wormholes.

5 Conclusions

Anomaly cancellation by the Green-Schwarz mechanism in the case of a certain $3+1$ dimension limit of a higher dimensioned string theory is questioned: We consider the gauge symmetry (needed to the $U(1)_X$ subgroup) that allegedly results from breaking a larger string theory gauge group using a field b derived from Kalb-Ramond B_{MN} that takes on a non-vanishing vacuum expectation value and thereby higgses the gauge field A_μ. This is manifested phenomenologically as an approximately conserved current without having the usual triangle summation anomaly requirement for avoiding gauge and mixed

anomalies. This is referred to as Green-Schwarz anomaly cancellation because the special way of having anomaly cancellations for string theory mass states is inherited to the $3+1$ dimensional limit.

If we have supersymmetry and such a charge with Green-Schwarz anomaly cancellation (effectively in the $3+1$ dimensions), then according to the calculations of Ref. [5] we have a Fayet-Iliopoulos D-term (16) which drives the dilaton field that is essentially in correspondence with the effective string coupling constant $g_s = \phi^2$ to be zero. That means that the theory becomes *free* and thus rather unrealistic. This is a severe trouble in itself for the models with a non-trivial anomaly cancellation mechanism.

However, contrary to this argument, if we should assume that in some mysterious way it were possible to get a string theory after all with supersymmetry surviving and having a significant D-term and string coupling in spite of such Green-Schwarz cancellation, then the world sheet instantons would prevent the breaking strength of the surviving global charge conservation from going to zero.

It is a major point of the present article that it is actually not achievable for the world sheet instantons do violate the $U(1)_X$-charge conservation. In this way we could avoid the mystery of having a gauge theory breaking from a spontaneous symmetry breaking, which does not break the current conservation. Provided that the world sheet instanton effects do not mysteriously cancel (which is though less safe to assume than naively expected), this mystery would disappear.

The authors working with the Green-Schwarz anomaly cancelling charges (usually) have in mind supersymmetric models. If the world sheet instanton effects would appear even in supersymmetric theories, the Green-Schwarz anomaly cancellation would become less suspicious of being pretended to behave strangely from the general point of view.

However, due to the Fayet-Iliopoulos D-term expected when we have such charges, the string coupling constant g_s may be driven to zero and the whole effect of world sheet instantons would disappear. This may though be not realistic because of bringing the whole string theory appears to be free.

If this kind of effect could work causing a breaking of the global charge conservation and thus avoiding some of the strangeness one may still seek (using the world sheet instantons) to declare a $U(1)_X$-charge needing the Green-Schwarz anomaly cancellation to have become less suspicious of being pretended to behave strangely from general point of view. However, one may still declare the way to be suspicious, in which the anomaly troubles are avoided in the energy range above the mass scale m of the gauge particle: The gauge fields are constrained to never have the configuration leading to anomalies!

If one gets a breaking of charge $U(1)_X$ due to the wormholes (or world sheet instantons), one could imagine as an interesting possibility to *use this breaking* instead of some spontaneous breaking induced from *e.g.* the Fayet-Iliopoulos D-term (as [9] proposes) to provide the soft breaking which is needed to use the charge $U(1)_X$ as a mass protecting charge to implement the large ratios of quark and lepton masses.

We further expressed our worry and suspicion whether such an electrodynamics which is constrained by $F_{\mu\nu}\widetilde{F}^{\mu\nu} = 0$ can really be considered as realistic on general physics grounds or whether it represents an unrealisable speculation concerning the relative orders of magnitude.

However, we thought there are reasons to believe that the wormholes would give violation of the $U(1)_X$ even above its Higgsing scale. That could mean that wormholes break the $U(1)_X$ symmetry completely under the use of $F_{\mu\nu}\widetilde{F}^{\mu\nu} = 0$. The point is that a very strong coupling of the b field causes that Coulomb field around the wormholes endings are not stable.

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